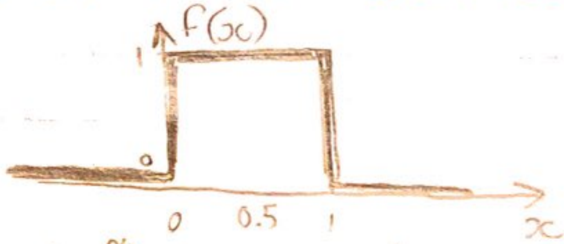
Refresher:

For a uniform PDF we have $f(x) = \begin{cases} 1 & \text{if } x \in [0,1] \\ 0 & \text{o.w.} \end{cases}$



$$E[X] = \int_{-\infty}^{\infty} dx \, x f(x) = \int_0^1 dx \, \frac{x^2}{2} = \left[\frac{x^2}{2}\right]_0^1 = 1/2$$

The realizations are uniformly distributed between 0 and 1

The average of these realizations is 0.5

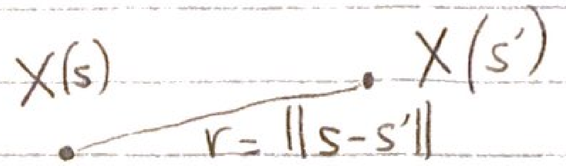
The center of mass of the PDF is at $x = 0.5$

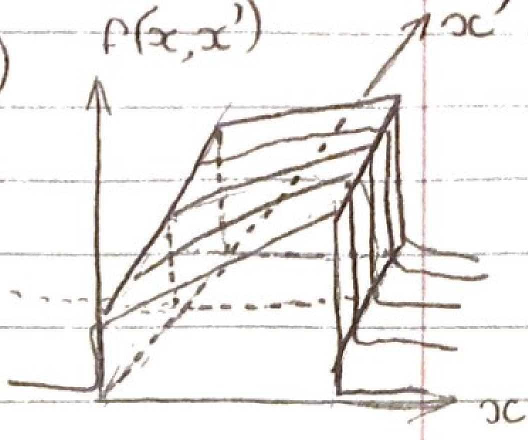
The value we expect (on average) is 0.5

$\boxed{\text{Make sure to plot the PDF before 0 and after 1}}$

Practice for HW7 part 1

$$f(x, x') = \begin{cases} 2(rx + x')/(r+1) & \forall x \in [0,1] \ \& \ x' \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$



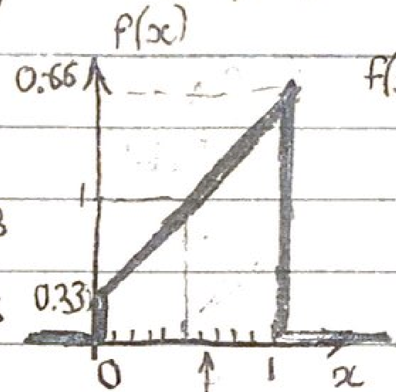


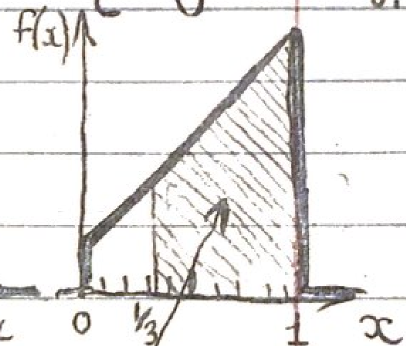
To estimate $X(s)$; If x' is not known, then use the marginal PDF $f(x)$. For example if $r=2$

$$f(x) = \int dx' \, f(x, x') = \begin{cases} (4x+1)/3 & \text{if } x \in [0\ 1] \\ 0 & \text{o.w} \end{cases}$$

To plot $f(x)$

x	$f(x)$
0	$1/3 = 0.333$
0.5	1
1	$5/3 = 1.666$





$m_x = \int dx \, x \, f(x) = \boxed{0.611}$ is the center of mass

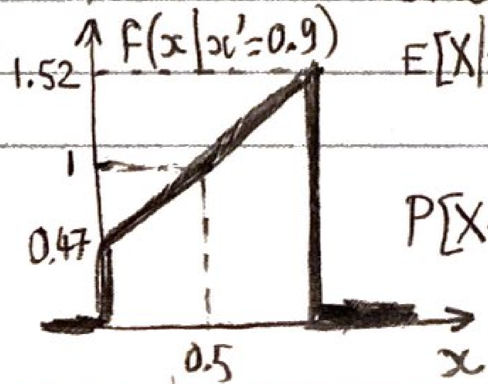
$P[X > 1/3] = \int_{1/3}^{1} dx \, f(x) = \boxed{0.8148}$ is the area under the curve

IF x' is known, then use $f(x|x')$ instead of $f(x)$

For example if $r=2$, and given $x' = 0.9$ then

$$f(x|x'=0.9) = \begin{cases} (20x+9)/19 & \text{if } x \in [0\ 1] \\ 0 & \text{o.w.} \end{cases}$$

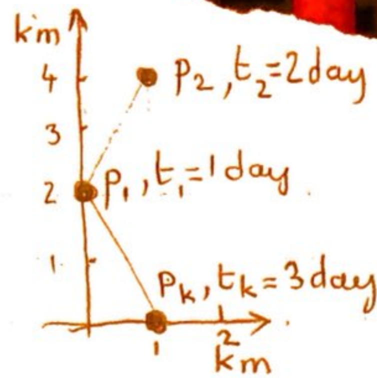
x	$f(x \mid x'=0.9)$
0	$9/19 = 0.47$
0.5	1
1	$29/19 = 1.52$



$E[X|x'=0.9] = \int dx \, x \, f(x|x=0.9)$
$= 0.5877$

$P[X < 1/2 \,|\, x'=0.9] = \int dx \, f(x|x'=0.9)$
$= 0.7836$

Practice for HW7 part 2



$p_1 = (0\,km, 2\,km, 1\,day)$

$p_2 = (1\,km, 4\,km, 2\,day)$

$p_k = (1\,km, 0\,km, 3\,day)$

The spatial lag r_{k1} between p_k and p_1 is

$$r_{k1} = \sqrt{(1-0)^2+(0-2)^2} = \sqrt{1+4} = \sqrt{5}\,km = 2.2361\,km$$

The time lag τ_{k1} between p_k and p_1 is

$$\tau_{k1} = |3-1| = 2\,day$$

The covariance c_{k1} between p_k and p_1 is

$$c_{k1} = c(r_{k1}, \tau_{k1}) = 1.5\exp\frac{-3(2.2361)}{1}\exp\frac{-3(2^2)}{3^2} + 0.5\exp\frac{-3(2.2361)}{30}\exp\frac{-3(2)}{700}$$

$$= 0.3969$$