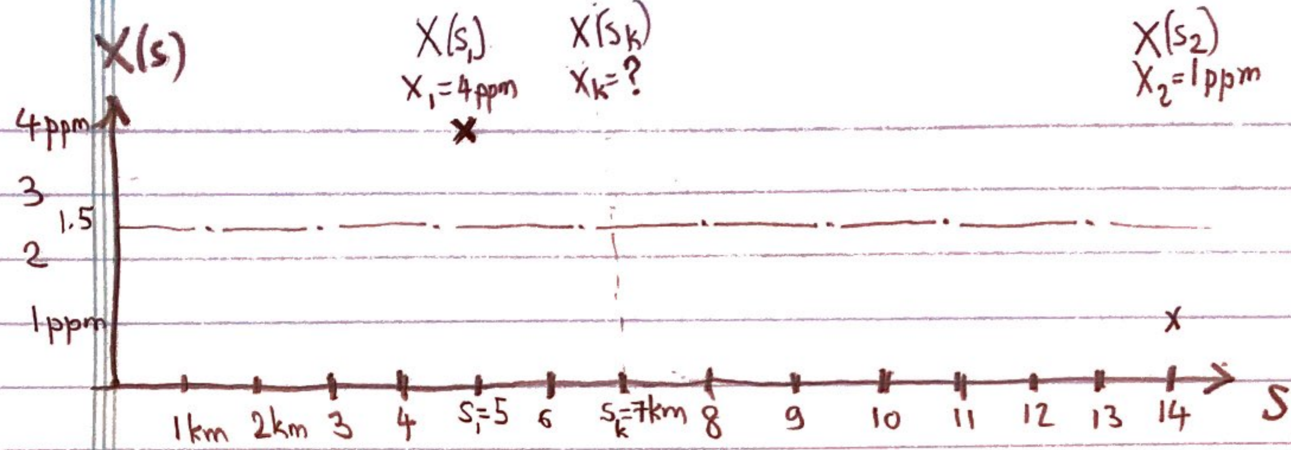


$f(x_1, x_k)$ is known

$m_1 = E[X_1] = \int dx_1 \int dx_k \; x_1 \; f(x_1, x_k)$

$m_k = E[X_k] = \quad \dots \quad x_k \quad \dots$

$c_{11} = cov(X_1, X_1) = \quad \dots \quad (x_1 - m_1)^2 \dots \qquad = var[X_1] = \sigma_{X_1}^2$

$c_{1k} = cov(X_1, X_k) = \quad \dots \quad (x_1 - m_1)(x_k - m_k) \dots$

$c_{kk} = cov(X_k, X_k) = \quad \dots \quad (x_k - m_k)^2 \dots \qquad = var[X_k] = \sigma_{X_k}^2$

$c_{k1} = cov(X_1, X_k) = \quad (x_k - m_k)(x_1 - m_1) \qquad = c_{1k}$

The moments are $\quad m_{1k} = \begin{bmatrix} m_1 \\ m_k \end{bmatrix} \qquad c_{1k,1k} = \begin{bmatrix} c_{11} & c_{1k} \\ c_{k1} & c_{kk} \end{bmatrix} = \begin{bmatrix} \sigma_{X_1}^2 & c_{1k} \\ c_{1k} & \sigma_{X_k}^2 \end{bmatrix}$

What is $X_k = ?$

- no data:

  $E[X_k] = m_k \qquad var_{X_k} = \sigma_{X_k}^2 \qquad$ 95% CI $= [m_k - 2\sigma_k \, , \, m_k + 2\sigma_k]$

- data $X_1 = 4$ ppm

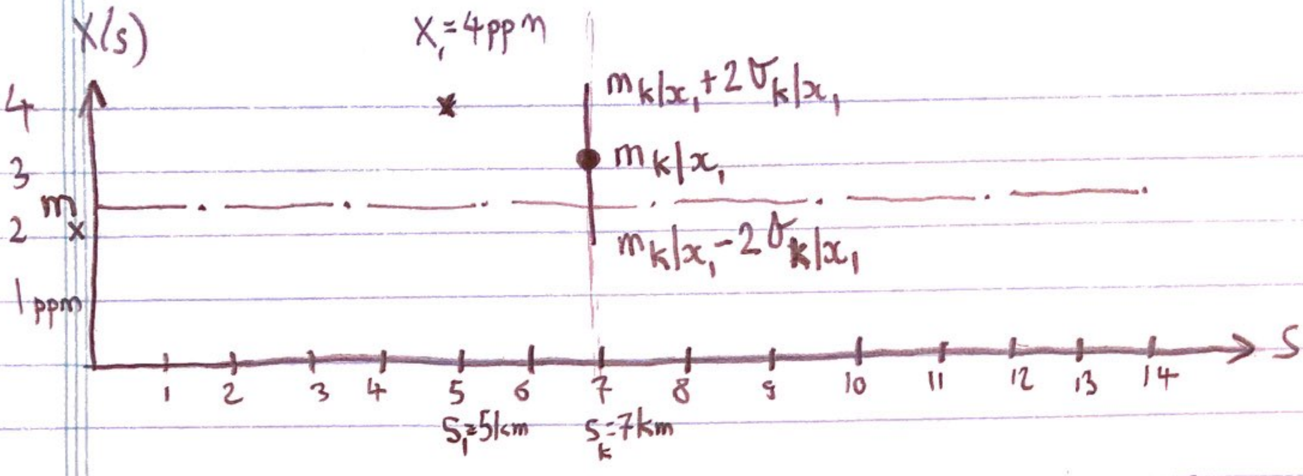
  derive $f(x_k | X_1 = x_1) = f(x_k, x_1) / f(x_k) = f(x_1, x_k) / \int dx_k \, f(x_1, x_k)$

  get $\quad f(x_k | X_1 = 4) = f(x_1, 4) / \int dx_k \, f(4, x_k)$

  $E[X_k | X_1 = 4] = \int dx_k \; x_k \; f(x_k | X_1 = 4) = m_{k|x_1=4}$

  $var[X_k | X_1 = 4] = \int dx_k \, (x_k - m_{k|x_1=4})^2 f(x_k | X_1 = 4) = \sigma_{k|X_1=4}$

  95% CI $= [m_{k|x_1} - 2\sigma_{k|x_1} \, , \, m_{k|X_1} + 2\sigma_{k|x_1}]$



$X(s)$ homogeneous with mean $m_x = 2.5$, $c(s,s') = \sigma_x^2 \exp \frac{-3r}{a_r}$
with $\sigma_x^2 = 1\,ppm^2$ $a_r = 5\,km$

kriging :

$E[X_k | X_1] = m_k + C_{1k}(C_{kk})^{-1}(x_1 - m_1)$

$var[X_k | X_1] = c_{kk} - c_{1k}(c_{kk})^{-1} c_{k1}$

$x_k = 7\,km$

$$m_{1k} = \begin{bmatrix} m_1 \\ m_k \end{bmatrix} = \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix} \qquad c_{1k,1k} = \begin{bmatrix} c_{11} & c_{1k} \\ c_{k1} & c_{kk} \end{bmatrix} = \begin{bmatrix} 1 & \exp\frac{-6}{5} \\ \exp\frac{-6}{5} & 1 \end{bmatrix}$$

$r_{11} = r(s_1, s_1) = \|s_1 - s_1\| = 0$

$c_{11} = c(s_1, s_1) = \sigma_x^2 \exp \frac{-3\|s_1 - s_1\|}{a_r} = \sigma_x^2 \exp \frac{-3(0)}{a_r} = \sigma_x^2 \exp(0) = \sigma_x^2 = 1$

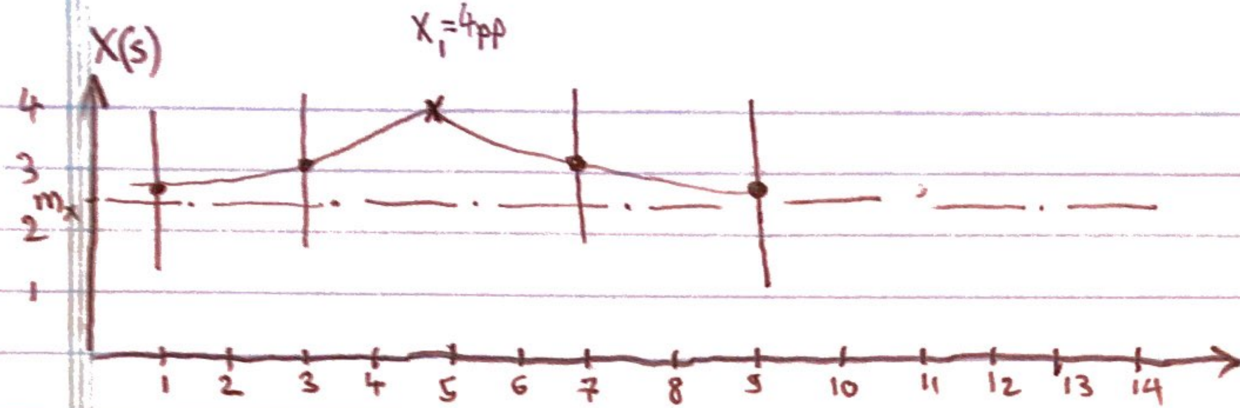
$r_{1k} = \|s_1 - s_k\| = 2\,km$

$c_{1k} = \sigma_x^2 \exp \frac{-3\|s_1 - s_k\|}{a_r} = \sigma_x^2 \exp \frac{-3(2)}{a_r} = 1\,ppm^2 \exp \frac{-3(2km)}{5km} = \exp \frac{-6}{5}$

$c_{kk} = \ldots = \sigma_x^2 = 1$

$m_{k|x_1} = m_k + (c_{1k} / c_{kk})(x_1 - m_1) = 2.5 + (\exp\frac{-6}{5} / 1)(4 - 2.5)$

$\sigma^2_{k|x_1} = c_{kk} - c_{1k}^2 / c_{kk} = 1 - (\exp\frac{-6}{5})^2 / 1$



$x_k = 9\ km$

$r_{11} = 0$, $r_{1k} = \|5km - 9km\| = 4km$ $r_{kk} = 0$

$c_{11} = \sigma_x^2 = 1\ ppm^2$

$c_{1k} = \sigma_x^2 \exp -\frac{3 r_{1k}}{a_r} = 1 \exp -\frac{3(4)}{5} = \exp -\frac{12}{4}$

$m_{k|x_1} = 2.5 + \left(\exp -\frac{12}{4} / 1\right)(4 - 2.5)$

$\sigma_{k|x_1} = 1 - \left(\exp -\frac{12}{4}\right)^2 / 1$