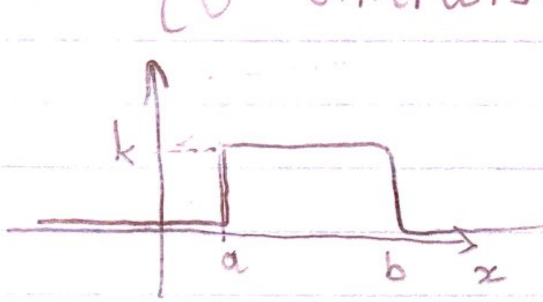
$$f(x) = \begin{cases} k & \text{if } x \in [a,b] \\ 0 & \text{otherwise} \end{cases}$$



example : $a=1, b=10$

$X = \{5, 1, 7, 10, 3, 8\}$

Normalization constraint:

$$\int_{-\infty}^{\infty} dx\, f(x) = 1 \Leftrightarrow \int_a^b dx\, k = 1 \Leftrightarrow [kx]_a^b = 1 \Leftrightarrow k(b-a) = 1 \Leftrightarrow k = 1/(b-a)$$

Expected value:

$$m_x = \int_{-\infty}^{\infty} dx\, x\, f(x) = \int_{-\infty}^{a} dx \cancel{(x \cdot 0)} + \int_a^b dx\, x/(b-a) + \int_b^{\infty} dx \cancel{(x \cdot 0)} = \frac{1}{(b-a)}\left[\frac{x^2}{2}\right]_a^b =$$

$$= \frac{1}{(b-a)} \frac{(b^2-a^2)}{2} = \frac{(\cancel{b-a})(b+a)}{(\cancel{b-a})\, 2} = \frac{b+a}{2}$$

Example: $X = \{5, 1, 7, 10, 3, 8\}$, $m_x \hat{=} \frac{1}{6}(5+1+7+10+3+8) = 5.67$

$$E[X] = \frac{a+b}{2} = \frac{1+10}{2} = 5.5$$

Variance:

$$V_x = \int_{-\infty}^{\infty} dx\, (x-m)^2 f(x) = \int_a^b dx \left(x - \frac{a+b}{2}\right)^2 \frac{1}{b-a} = \dots = \frac{(b-a)^2}{12}$$

or

$$V_x = \int_{-\infty}^{\infty} dx\, (x^2 - 2xm + m^2) f(x) = \int_{-\infty}^{\infty} dx\, x^2 f(x) - 2m \underbrace{\int_{-\infty}^{\infty} dx\, x\, f(x)}_{m} + m^2 \underbrace{\int_{-\infty}^{\infty} dx\, f(x)}_{1}$$

$$\boxed{Var = \int_{-\infty}^{\infty} x^2 f(x) - m^2} = \int_a^b dx\, \frac{x^2}{(b-a)} - m^2 = \left[\frac{x^3}{3(b-a)}\right]_a^b - m^2 = \frac{b^3-a^3}{3(b-a)} - \frac{(a+b)^2}{4} = \dots$$