$$f(x,y)=\begin{cases}1+x-2xy & \forall x\in[0,1]\ \forall y\in[0,1]\\ 0 & \text{otherwise}\end{cases}$$

Normalization: $E[1]=\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\ f(x,y)=\int_0^1 dx\int_0^1 dy(1+x-2xy)=\int_0^1 dx\left[y+xy-2x\frac{y^2}{2}\right]_{y=0}^{y=1}=\int_0^1 dx(1+x-x)=\int_0^1 dx\,1=[x]_0^1=1$

d: $E[X]=\int_0^1 dx\int_0^1 dy\ x(1+x-2xy)=\int_0^1 dx\int_0^1 dy\left(x+x^2-2x^2y\right)=\int_0^1 dx\left[xy+x^2y-2x^2\frac{y^2}{2}\right]_{y=0}^{y=1}=\int_0^1 dx\left(x+x^2-x^2\right)=\left[\frac{x^2}{2}\right]_0^1=\frac{1}{2}$

e: $E[Y]=\int_0^1 dx\int_0^1 dy\ y(1+x-2xy)=\int_0^1 dx\int_0^1 dy\left(y+xy-2xy^2\right)=\int_0^1 dx\left[\frac{y^2}{2}+x\frac{y^2}{2}-2x\frac{y^3}{3}\right]_{y=0}^{y=1}=\int_0^1 dx\left(\frac{1}{2}+\frac{x}{2}-\frac{2}{3}x\right)=\left[\frac{x}{2}+\frac{x^2}{4}-\frac{2x^2}{3\times2}\right]_0^1=\frac{1}{2}+\frac{1}{4}-\frac{1}{3}=\frac{5}{12}$

f: var_X,

$E[X^2]=\int_0^1 dx\int_0^1 dy\ x^2(1+x-2xy)=\int_0^1 dx\int_0^1 dy\left(x^2+x^3-2x^3y\right)=\int_0^1 dx\left[x^2y+x^3y-2x^3\frac{y^2}{2}\right]_{y=0}^{y=1}=\int_0^1 dx\left(x^2+x^3-x^3\right)=\left[\frac{x^3}{3}\right]_0^1=\frac{1}{3}$

$var_X=E[X^2]-m_X^2=\frac{1}{3}-\left(\frac{1}{2}\right)^2=\frac{4-3}{12}=\frac{1}{12}$

g: var_Y

$E[Y^2]=\int_0^1 dx\int_0^1 dy\ y^2(1+x-2xy)=\int_0^1 dx\int_0^1 dy\left(y^2+xy^2-2xy^3\right)=\int_0^1 dx\left[\frac{y^3}{3}+x\frac{y^3}{3}-2x\frac{y^4}{4}\right]_{y=0}^{y=1}=\int_0^1 dx\left(\frac{1}{3}+\frac{x}{3}-\frac{x}{2}\right)=\int_0^1 dx\left(\frac{1}{3}-\frac{x}{6}\right)=\left[\frac{x}{3}-\frac{x^2}{12}\right]_0^1=\frac{1}{3}-\frac{1}{12}=\frac{3}{12}=\frac{1}{4}$

$var_Y=\frac{1}{4}-\left(\frac{5}{12}\right)^2=\frac{1}{4}-\frac{25}{144}=\frac{36-25}{144}=\frac{11}{144}$

h: $cov(X,Y)$

$E[XY]=\int_0^1 dx\int_0^1 dy\ xy(1+x-2xy)=\int_0^1 dx\int_0^1 dy\left(xy+x^2y-2x^2y^2\right)=\int_0^1 dx\left[(x+x^2)\frac{y^2}{2}-2x^2\frac{y^3}{3}\right]_{y=0}^{y=1}=\int_0^1 dx\left(\frac{x}{2}+\frac{x^2}{2}-\frac{2}{3}x^2\right)=\int_0^1 dx\left(\frac{x}{2}-\frac{x^2}{6}\right)=\left[\frac{x^2}{4}-\frac{x^3}{18}\right]_0^1=\frac{1}{4}-\frac{1}{18}=\frac{9-2}{36}=\frac{7}{36}$

$cov(X,Y)=\frac{7}{36}-\frac{1}{2}\times\frac{5}{12}=\frac{7}{36}-\frac{5}{24}=\frac{14-15}{72}=\frac{-1}{72}$

i: $\rho(X,Y)=\frac{cov(X,Y)}{\sigma_X\sigma_Y}=\frac{cov(X,Y)}{\sqrt{var_X}\sqrt{var_Y}}=\frac{-1}{72}\times\frac{1}{\sqrt{\frac{1}{12}}\sqrt{\frac{11}{144}}}=\frac{-\sqrt{12}\sqrt{144}}{72\sqrt{11}}=\frac{-\sqrt{12}\ 12}{6\times12\times\sqrt{11}}=-\frac{\sqrt{12}}{6\sqrt{11}}\simeq-0.1741$

$$f(x,y) = \begin{cases} 1+x-2xy & \forall x \in [0,1] \text{ AND } \forall y \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

$$j: f_Y(y) = \int_{-\infty}^{\infty} dx\, f(x,y) = \int_{-\infty}^{0} dx\, f(x,y) + \int_{0}^{1} dx\, f(x,y) + \int_{1}^{\infty} dx\, f(x,y) = \int_{-\infty}^{0} dx\, 0 + \int_{0}^{1} dx\, f(x,y) + \int_{1}^{\infty} dx\, 0 = \begin{cases} \int_0^1 dx\,(1+x-2xy) = \left[x + \frac{x^2}{2} - \frac{2x^2}{2}y\right]_{x=0}^{x=1} = \frac{3}{2} - y & \forall y \in [0,1] \\ 0 & \text{if } y \notin [0,1] \end{cases}$$

$$f_Y(y) = \begin{cases} 3/2 - y & \forall y \in [0,1] \\ 0 & \text{o.w.} \end{cases}$$

k: $f_{X|Y}(x|Y=y)$ can only be define for $y \in [0,1]$ since $P[y<0]=0$ and $P[y>1]=0$, i.e. y cannot be outside $[0,1]$

If $y \in [0,1]$ then
$$f_{X|Y}(x|Y=y) = \frac{f(x,y)}{f_Y(y)} = \frac{f_{XY}(x,y)}{3/2 - y} = \begin{cases} \frac{1+x-2xy}{\frac{3}{2}-y} & \text{if } x \in [0,1] \,\&\, y \in [0,1] \\ 0 & \text{if } x \notin [0,1] \,\&\, y \in [0,1] \end{cases}$$

$$l.\ P[X<1|Y=0] = \int_{-\infty}^{1} dx\, f_{X|Y}(x|Y=0) = \int_{-\infty}^{0} dx\, f_{X|Y}(x|Y=0) + \int_{0}^{1} dx\, f_X(x|Y=0) = \int_{-\infty}^{0} dx\, 0 + \int_0^1 dx\, \frac{2}{3}(1+x) = \frac{2}{3}\left[x + \frac{x^2}{2}\right]_0^1 = \frac{2}{3} \times \frac{3}{2} = 1$$

$$m.\ E[X|Y=0] = \int_{-\infty}^{\infty} dx\, x f_{X|Y}(x|Y=0) = \int_0^1 dx\, \frac{2}{3}(x+x^2) = \frac{2}{3}\left[\frac{x^2}{2} + \frac{x^3}{3}\right]_0^1 = \frac{2}{3}\left(\frac{1}{2} + \frac{1}{3}\right) = \frac{2}{3} \times \frac{5}{6} = \frac{5}{9}$$

$$n.\ E[X|Y=1] = \int_{-\infty}^{\infty} dx\, x f_{X|Y}(x|Y=1) = \int_0^1 dx\, x\, 2(1-x) = 2\int_0^1 dx\,(x - x^2) = 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left(\frac{1}{2} - \frac{1}{3}\right) = 2\,\frac{1}{6} = \frac{1}{3}$$