

$$f_{xy}(x,y) = \begin{cases} k & \text{if } x \in [a,b] \text{ \& } y \in [a,b] \\ 0 & \text{o.w.} \end{cases}$$

$$1) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{xy}(x,y) = 1 \iff \int_a^b \int_a^b k = 1 \iff \int_a^b [ky]_{y=a}^{y=b} = 1$$

$$\iff \int_a^b k(b-a) = 1 \iff k(b-a) \left[x \right]_a^b = 1$$

$$\iff k(b-a)(b-a) = 1 \iff k = \frac{1}{(b-a)^2}$$

$$2) m_x = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dy \, x f(x,y) = \int_a^b \int_a^b dy \, \frac{x}{(b-a)^2} = \frac{1}{(b-a)^2} \int_a^b [xy]_{y=a}^{y=b}$$

$$= \frac{1}{(b-a)^2} \int_a^b x(b-a) = \frac{1}{(b-a)} \left[\frac{x^2}{2} \right]_a^b = \frac{1}{2(b-a)} (b^2 - a^2) = \frac{1}{2} \frac{(a+b)(a-b)}{(a-b)}$$

$$m_x = \frac{a+b}{2}$$

$$3) E[X^2] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dy \, x^2 f(x,y) = \int_a^b \int_a^b dy \, \frac{x^2}{(b-a)^2} = \frac{1}{(b-a)^2} \int_a^b [x^2 y]_{y=a}^{y=b}$$

$$= \frac{(b-a)}{(b-a)^2} \left[\frac{x^3}{3} \right]_{x=a}^{x=b} = \frac{1}{(b-a)} \frac{(a^3 - b^3)}{3} = \frac{(a-b)(a^2 + ab + b^2)}{3(a-b)}$$

$$E[X^2] = \frac{a^2 + ab + b^2}{3}$$

$$\text{Var}_x = E[X^2] - m_x^2 = \frac{a^2}{3} + \frac{ab}{3} + \frac{b^2}{3} - \frac{(a+b)^2}{4} = \frac{a^2}{3} + \frac{ab}{3} + \frac{b^2}{3} - \frac{a^2}{4} - \frac{ab}{2} - \frac{b^2}{4}$$

$$= \frac{a^2}{12} - \frac{ab}{6} + \frac{b^2}{12} = \frac{a^2 - 2ab + b^2}{12} = \frac{(b-a)^2}{12}$$

$$\text{Var}_x = \frac{(b-a)^2}{12}$$

$$4) E[Y] = \int_a^b dx \int_a^b dy \frac{y}{(b-a)^2} = \int_a^b dx \left[\frac{y^2}{2} \right]_a^b = \frac{a+b}{2(b-a)} \left[x \right]_a^b = \frac{a+b}{2}$$

$$5) E[XY] = \int_a^b dx \int_a^b dy \frac{xy}{(b-a)^2} = \int_a^b dx \left[\frac{xy^2}{2} \right]_{y=a}^{y=b} = \frac{(a+b)}{2(b-a)} \left[\frac{x^2}{2} \right]_a^b = \frac{(a+b)^2}{4}$$

$$\text{cov}[XY] = E[XY] - E[X]E[Y] = \frac{(a+b)^2}{4} - \frac{(a+b)}{2} \frac{(a+b)}{2} = 0$$

$$\boxed{\text{cov}[XY] = 0}$$