Calculus

Exercise 1.

$$\int_0^1 dy \int_0^1 dx\,(2x+3y^2)$$

$$= \int_0^1 dy \left\{ \int_0^1 dx\,(2x+3y^2) \right\}$$

$$= \int_0^1 dy \left\{ \left[2\frac{x^2}{2} + 3y^2 x \right]_{x=0}^{x=1} \right\}$$

$$= \int_0^1 dy \left\{ \left[2\frac{1^2}{2} + 3y^2(1) \right] - \left[2\frac{0^2}{2} + 3y^2 0 \right] \right\}$$

$$= \int_0^1 dy \left\{ 1 + 3y^2 \right\}$$

$$= \int_0^1 dy\,(1+3y^2)$$

$$= \left[y + 3\frac{y^3}{3} \right]_{y=0}^{y=1}$$

$$= \left(1 + 3\frac{(1)^3}{3}\right) - \left(0 + 3\frac{0^3}{3}\right) = 1+1 = 2$$

$f(x)$	$\int f(x)$
x^n	$x^{n+1}/(n+1)$
x^1	$x^2/2$
x^2	$x^3/3$
a	ax
x	$x^2/2$
$a = ax^0$	ax
y	yx
$3y^2$	$3y^2 x$
$x + x^2$	$x^2/2 + x^3/3$
$2x+3y^2$	$2x^2/2 + 3y^2 x$

$F(y)$	$\int f(y)$
y^n	$y^{n+1}/n+1$
y^1	$y^2/2$
y^2	$y^3/3$
a	ay
x	xy
$a = ay^0$	ay
$2x$	$2xy$
$3y^2$	$3y^3/3$

Check

$$\int_0^1 dx \left\{ \int_0^1 dy\,(2x+3y^2) \right\}$$

$$= \int_0^1 dx \left\{ \left[2xy + 3y^3/3 \right]_{y=0}^{y=1} \right\}$$

$$= \int_0^1 dx \left\{ (2x+1) - (0+0) \right\} = \int_0^1 dx\,(2x+1)$$

$$= \left[2x^2/2 + x \right]_{x=0}^{x=1} = (1+1) - (0+0) = 2$$