Homework 4

Random variables and spatial covariance models

Date given: 9/23

Due: 9/30 noon

Part 1 (20 points)

Consider the following function

$$f_X(x) = \begin{cases} kx, & \forall x \in [0,2] \\ 0, & o.w. \text{(otherwise)} \end{cases}$$

a. Determine the constant k to ensure that the function $f_X(x)$ is the pdf of a random variable x (i.e. apply the normalization constraint).

b. Calculate the expected value of X (i.e. calculate $m_X = E_X[X]$)

c. Calculate the variance of X (i.e. calculate $\text{var}_X = E_X[(X-m)^2]$)

d. Calculate the probability that $0.1 < X < 0.3$

Part 2 (45 points)

Two random variables X and Y have the following bivariate pdf:

$$f_{XY}(x,y) = \begin{cases} \frac{1}{5}(x+2y), \forall x \in [0,1] \text{ and } y \in [0,2] \\ 0, \quad o.w. \end{cases}$$

e. Calculate the mean of X, $m_X = E_{XY}[X] = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \; x \, f_{XY}(x,y)$

f. Calculate the mean of Y, $m_Y = E_{XY}[Y]$

g. Calculate the variance of X, $\text{var}_X = E_{XY}[(X - m_X)^2]$.

h. Calculate the variance of Y, var_Y

i. Calculate the covariance between X and Y, $\text{cov}(X,Y)$

j. Calculate the correlation between X and Y, $\rho_{XY} = \text{cov}(X,Y) / \sqrt{\text{var}_X \; \text{var}_Y}$

k. Calculate the marginal pdf of $f_{XY}(x,y)$ with respect to y

l. Calculate conditional pdf of X given $Y=y$

m. Calculate probability that $X<0.5$ given that $Y=2$

Part 3 (35 points)

Describe in details two models for the covariance $c_X(r)$ for a homogeneous spatial random field $X(\mathbf{s})$. Provide the equation for the covariance models. Explain the type of spatial variability described by that model. Explain what each parameter is.

Hints:

For each covariance model there should be one section with the overall description of the model, and one section for each parameters. For example if a model has two parameters, then there should be one section providing an overall description of that model, one section describing the first parameter, and one section describing the second parameter.

The section providing the overall description of a model should be as follow: Provide the equation $c_X(r)$ of the model, where r is the spatial lag. Show a graph of the equation, that is show how $c_X(r)$ changes with respect to the variable r. Describe in words the shape of the graph (in particular the shape of $c_X(r)$ with respect

of r when r is small, also called the shape at the origin). Give a qualitative description the type of spatial variability (e.g. smooth versus random) that the covariance model describes, and why that is. For example, there might be a covariance model $c_X(r)$ with a given shape at the origin that describes a field $X(\mathbf{s})$ with smooth spatial variability, while there might be another covariance model with another shape at the origin describing a spatial variability that is less smooth. Finally name each parameter involved in the covariance model.

The section describing a given parameter should explain how that parameter changes the shape of the covariance model (i.e. the shape of the graph of the covariance model), and it should describe how that change in the shape of the covariance model affects the corresponding spatial variability of the spatial field $X(\mathbf{s})$. If done well, this section should allow a practitioner to select a proper parameter value based on either (a) a map of the data, or (b) a graph of experimental covariance values.

You will be graded based on the following: The quality of your description of the covariance models and the corresponding spatial variability described by these models. The extent to which your text makes sense to a reader who does not know geostatistics. The extent to which your text would allow a practitioner to select the proper model and the proper parameter values based on maps of the data or on the graph of experimental covariance values.

Save your homework as a well-written word document named yourfirstname_yourlastname_hwk4.docx and send it to the TA. For parts 1 and 2 provide all your derivations and results either directly typed in the word document, or as a scanned pdf file.